An Efficient Anonymous Credentials System

Jan Camenisch
IBM Research
joint work w/ Anna Lysyanskaya, Ivan Damgård, Victor Shoup

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Outline

I. Requirements of Anonymous Credential System

II. Abstract Solution

III. The Technical Bit
   Signature Scheme
   Commitments and Proof Protocols
   Encryption Scheme
The Problem: Pseudonym System

[Chaum85]
The Problem, Even Larger: Extended Pseudonym System

Driver's License

Insurance

Dangerous Cars

Judge

Cryptography for Privacy -- Credential\(^+\) Systems
Basic Requirements of Pseudonym System

- Protection of user's privacy
  - anonymity
  - unlinkeability (multi-use)

- Unforgeability of credentials

- Consistency of credentials (no pooling)
Extra Requirements of Pseudonym System

- Sharing of credentials
- Anonymity revocation
  - local
  - global
- Revocation of credentials
- Encoding of attributes
- One-show credential (e-cash)
  - off-line & on-line
- k-spendable credentials
- ......
Some History

- Chaum ’85: introduced scenario
- Chaum & Evertse ’87: solution based on a semi-trusted party
- Damgård ’90: theoretical solution
- Brands ’95–’99: one-show credentials with different attributes
- LRSW ’99: practical solution for one-show credentials
- Camenisch-Lysyanskaya ’00: efficient multi-show w/ attributes
- Verheul ’01: bi-linear map multi-show
- Camenisch-Lysyanskaya ’04: Discrete log based.

Special cases: e-cash, group signatures, identity escrow
Proving Ownership Solution
Proving Ownership Solution
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Proving Ownership Solution
Proving Ownership Vs. Using Blind Signatures

Certificates can be used *multiple* times!

Certificates can be used only *once*!
Required Technologies

Signature Schemes

Encryption Schemes

Commitment Schemes

Zero-Knowledge Proofs

..... challenge is to do all this efficiently!
Zero-Knowledge Proofs of Knowledge of Discrete Logarithms

Given group \( \langle g \rangle \) and element \( y \in \langle g \rangle \).

Prove knowledge of \( x = \log g y \) such that verifier only learns \( y \) and \( g \).

Prover:

\[ t := g^r \]

Verifier:

\[ t = g^s y^c \]

**PK\{(\alpha): y = g^\alpha\}**
Zero Knowledge Proofs II

Non-interactive (Fiat-Shamir heuristic):

$$\text{PK}\{(\alpha): \ y = g^\alpha \}(m)$$

Logical combinations:

$$\text{PK}\{(\alpha,\beta): \ y = g^\alpha \land z = g^\beta \land u = g^\beta h^\alpha \}$$
$$\text{PK}\{(\alpha,\beta): \ y = g^\alpha \lor z = g^\beta \}$$

Intervals and groups of different order (under SRSA):

$$\text{PK}\{(\alpha): \ y = g^\alpha \land \alpha \in [A,B] \}$$
$$\text{PK}\{(\alpha): \ y = g^\alpha \land z = g^\alpha \land \alpha \in [0,\min\{\text{ord}(g),\text{ord}(g)\}] \}$$
Commitment Schemes

Group \( G = \langle g \rangle = \langle h \rangle \) of order \( q \)

To commit to element \( x \in \mathbb{Z}_q \):

- perfectly hiding, computationally binding (Pedersen):
  
  choose \( r \in \mathbb{Z}_q \) and compute \( c = g^x h^r \)

- computationally hiding, perfectly binding:
  
  choose \( r \in \mathbb{Z}_q \) and compute \( c = (g^x h^r, g^r) \)

To commit to integer \( x \in \mathbb{Z} \) (Damgård, Fujisaki):

- similarly, if order of \( G \) is not known, e.g., \( G = QR_n \)
The Strong RSA Assumption

Flexible RSA Problem: Given RSA modulus $n$ and $z \in QR_n$ find integers $e$ and $u$ such that

$$u^e = z \mod n$$

- Introduced by Barić & Pfitzmann '97 and Fujisaki & Okamoto '97
- Hard in generic algorithm model [Damgård & Koprowski '01]
Signature Scheme based on the SRSA Assumption I

Public key of signer: RSA modulus $n$ and $a_i, b, d \in QR_n$.

Secret key: factors of $n$.

To sign $k$ messages $m_1, ..., m_k \in \{0,1\}^\ell$:

- choose random prime $e > 2^\ell$ and integer $s \approx n$
- compute $c$ such that

$$d = a_1^{m_1} \cdot ... \cdot a_k^{m_k} \cdot b^s \cdot c^e \mod n$$

- signature is $(c,e,s)$
Signature Scheme based on the SRSA Assumption II

A signature \((c,e,s)\) on messages \(m_1, \ldots, m_k\) is valid iff:

- \(m_1, \ldots, m_k \in \{0,1\}^\ell\)
- \(e > 2^\ell\)
- \(d = a_1^{m_1} \cdots a_k^{m_k} b^s \cdot c^e \mod n\)

Theorem: Signature scheme is secure against adaptively chosen message attacks under SRSA assumption.
Getting a Signature on a Secret Message

$C = a_1^{sk} b^{s'}$
Getting a Signature on a Secret Message

\[ C = a_1^{sk} b^{s'} \]

\[ \text{PK}(\mu_1, \sigma') : C = a_1^{\mu_1} b^{\sigma'} \]
Getting a Signature on a Secret Message

C = a_{1}^{sk} b^{s'}

PK{(\mu_{1}, \sigma')} : C = a_{1}^{\mu_{1}} b^{\sigma'}

d = C a_{2}^{nym} b^{s''} c^{e} \mod n

d = a_{1}^{sk} a_{2}^{nym} b^{s''} + s' c^{e} \mod n
Proof of Knowledge of a Signature

Observe:

- Let $c' = c b^{s'} \mod n$ with randomly and $s'$
- then $d = c'^e a_1^{m_1} \cdot ... \cdot a_k^{m_k} b^{s*} \pmod{n}$,
  i.e., $(c', e, s*)$ is also a valid signature!

Therefore, to prove knowledge of signature on some $m$

- provide $c'$
- $PK\{(\varepsilon, \mu_1, ..., \mu_k, \sigma) : \quad d := c'^\varepsilon a_1^\mu_1 \cdot ... \cdot a_k^\mu_k b^\sigma$
  $\land \mu_1 \in \{0,1\}^\ell \land \varepsilon \in 2^{\ell+1} \pm \{0,1\}^\ell \}$
Proof of Knowledge of a Signature

Using second Commitment

- $C = a_1^{sk} b^{s^*}$

To prove knowledge of signature on some $m$

- provide $c'$

- $PK\{(\varepsilon, \mu_1, ..., \mu_k, \sigma, \sigma^*, ) :$

\[ C = a_1^{\mu_1} b^{\sigma^*} \land d := c' \varepsilon a_1^{\mu_1} \cdots a_k^{\mu_k} b^\sigma \}$
Verifiable Encryption
The Decision Composite Residuosity Assumption

The DCR Problem: Given \( n \) and \( x \), decide whether or not
\[
x \in (\mathbb{Z}_n^*)^n.
\]

- Introduced by Paillier '99.
- If \( n = (2p' + 1)(2q' + 1) \) then \( \mathbb{Z}^*_n = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_n \times \mathbb{Z}_{p'q'} \).
- \((1+n)^u = (1+un) \mod n^2\).
An Encryption Scheme

Public Key:  \( n \) and  \( g, Y_1, Y_2, Y_3 \in \langle (g')^{2n} \rangle \), where  \( g' \in \mathbb{Z}^*_{n^2} \).

Secret Key:  \( x_i = \log Y_i \)

Encryption message  \( m \in [0,n] \) under label  \( L \):

- choose  \( r \in [0,n/4] \)

- \( u := g^r, e := Y_1^r (1+n)^m, v := \text{abs}(Y_2 Y_3^{H(u,e,L)})^r \)

- output  \((u,e,v)\).

where  \text{abs()} \space maps  \((a \mod n^2)\) to  \((n^2 - a \mod n^2)\) if  \(a > n^2/2\),

and  \((a \mod n^2)\) otherwise, where  \(0 < a < n^2\).
An Encryption Scheme

Decryption of ciphertext \((u,e,v)\) under label \(L\):

- verify \(v = \text{abs}(v)\) and \(u^{2(x_2 + H(u,e,L)x_3)} = v^2\).
- \(\check{c} := (e/u^{x_1})^{2^t}\) where \(t = 2^{-1} \mod n\),
- if \(n \mid (\check{c}-1)\) output \(m := (\check{c}-1)/n\), otherwise output \(\bot\).

Intuition: remember \((1+n)^q = 1+an \pmod{n^2}\)

so \((e/u^{x_1}) = \gamma_1 \left(1+n\right)^m / (g^r)^{x_1} = (1+n)^m = 1+mn\)

Theorem: Encryption scheme is secure against adaptively chosen ciphertext attacks under DCR assumption.
Verifiable Encryption of a Discrete Logarithm

Let \( d = a_1^{sk} a_2^{nym} b^s c^e \pmod{n} \) be a driver's license and \((u,v,e)\) be an encryption of \(nym\).

To prove that \((u,v,e)\) indeed encrypts \(m\):

\[
PK\{(\epsilon, \mu_1, \mu_2, \rho, \sigma) : \\
d := c^\epsilon a_1^{\mu_1} a_2^{\mu_2} b^\sigma \land \mu_1, \mu_2 \in \{0,1\}^\ell \land \\
u^2 = g^{2\rho} \land e^2 = Y_1^{2\rho} (1+n)^{2\mu_2} \land v^2 = (Y_2 Y_3^{H(u,e,L)})^{2\rho} \}
\]
Conclusion & Outlook

- Efficient Anonymous Credentials and more!
- TCG TPM V1.2 will have some of this

  *Was known in theory; soon your computer will have it.*

- EU Project PRIME will have all of this

  **www.prime-project.eu.org**

- Plans:
  - Open source
  - Lots of more research :-}
Thanks for your attention!